Answer to the Referee remarks on the NF-100275 paper

"Helical modulation of the electrostatic potential due to magnetic islands in toroidal plasma confinement devices", by G.Ciaccio et al.

We thank the Referee for appreciating our work, and for his detailed comments. We hope we have clarified the various issues of clarity the Referee raised. Here follows a point-to-point reply.

1. Many points raised by the Referee, namely 1),2),5),8), 11) and 15), show that the main message of the paper was not conveyed in the proper way. We decided then to extend the Letter to a regular paper, move the experimental part at the beginning, after the Introduction, and clarify the context in which the specific simulations with ORBIT have been performed for this paper. The context is that of the stellarator, where non-ambipolar fluxes arise easily from strong banana transport, and must be balanced by an ambipolar electric field. In this context, δf Monte Carlo codes are customary: some examples include FORTEC-3D [1, 2] and EUTERPE-GSRAKE [3]. Alternatively, one can use a linearized drift-kinetic equation, such as in the case of DKES-PENTA [4] and the analytical calculations by Shaing *et al.* [5, 6]. A good review of the impact of a 3D field on E^r has been presented by Callen at the IAEA Conference in Daejeon [7].

   In the case of chaotic structures, where tiny details of stochastic layers and fixed points dominate over neoclassical effects in driving transport, an optimal tool is a guiding-center code, such as ORBIT [8], which has the additional capability of describing collisional effects for electrons/ions, through a Monte-Carlo package based on the Boozer-Kuo operator [9]. In this sense, the reference to the work by Nardon and Valerie Izzo is inappropriate, since JOREK calculates an MHD flow, which is clearly a different physical object: MHD cannot account for two-fluid effects. In fact, in RFX, the MHD flow is three orders of magnitude smaller than the edge flow which is related to the ambipolar potential [10].

   It is finally worth noting that ORBIT is routinely used for diverted tokamaks [11, 12], and that the inclusion of an inductive correction for fast rotating islands is not a problem, and has already been done, e.g. on ASDEX [13].

   Therefore, the choice of the code ORBIT, the device TEXTOR and a fixed island is motivated by the nature of the physical problem, the available measurements, and by simplicity: we do not see the way a D-shaped equilibrium, or a rotating island (which can be anyway included in ORBIT), should significantly alter the main result, i.e. that a potential develops with the same symmetry as the parent island. There are a number of ways to demonstrate this: a very simple, heuristic argument can be derived by considering the GC equations of motion. The equations are provided in Roscoe's White book [14] or, more recently, in [15]. Specify the equation for electrons and ions, and neglect the ripple:

$$\dot{P}_\zeta^{(e)} = \rho_\parallel^e B^2 \; \partial_\zeta \alpha + \frac{\partial \Phi}{\partial \zeta} \tag{1}$$

$$\dot{P}_\zeta^{(i)} = \rho_\parallel^i B^2 \; \partial_\zeta \alpha - \frac{\partial \Phi}{\partial \zeta} \; , \tag{2}$$

where $\rho_\parallel = m v_\parallel / eB$ is the "parallel" gyro-radius, P_ζ is the canonical toroidal momentum, and the magnetic field perturbation is treated as $\delta \vec{B} = \nabla \times \alpha \vec{B}_0$. Flux coordinates of Boozer-type (ψ_p, θ, ζ) are used. The meaning of Eqs. (1-2) is that, in presence of a 3D field α, the toroidal momentum P_ζ is no more conserved in time. On the other hand, a larger drift (larger $\rho_\parallel^i$) for ions determines a different response to the symmetry breaking brught about by $\partial_\zeta \alpha$, and this different change in $\dot{P}_\zeta$ must be balanced by the ambipolar potential Φ. Subtract (1) from (2)

$$\dot{P}_\zeta^{(i)} - \dot{P}_\zeta^{(e)} = (\rho_\parallel^i - \rho_\parallel^e) B^2 \; \partial_\zeta \alpha - 2 \frac{\partial \Phi}{\partial \zeta} = 0 \; , \tag{3}$$

and solve in terms of the potential

$$\frac{\partial \Phi}{\partial \zeta} = \frac{1}{2} \, (\rho_\parallel^i - \rho_\parallel^e) B^2 \; \partial_\zeta \alpha \; . \tag{4}$$

If α is a single mode, $\alpha = \alpha_{m,n} \sin(m\theta - n\zeta + \phi)$, with ϕ phase of the mode, Eq. (4) can be integrated to give

$$\begin{aligned} \Phi(\psi_p, \theta, \zeta) =& \Phi_0(\psi_p) + \frac{1}{2} \, (\rho_\parallel^i - \rho_\parallel^e) B^2 \alpha_{m,n}(\psi_p) \sin(m\theta - n\zeta + \phi) \\ =& \Phi_0(\psi_p) + \frac{1}{2} \, (\rho_\parallel^i - \rho_\parallel^e) B^2 \alpha_{m,n}(\psi_p) \sin u \; , \end{aligned} \tag{5}$$

where u is the helical angle. Of course, our simplified equations do not catch the overall complexity of the electron and ion motion, but they show that, whenever you break the symmetry, this is done differently for electrons and ions, and a balancing Φ is needed, which will be modulated as $\sin u$. This is independent of the shape of the equilibrium flux surfaces ψ_p, and it is valid also for slowly rotating islands, where $u = m\theta - n\zeta + \omega t$.

In the case of *fast* rotating islands (in TEXTOR, with frequency > 3 kHz), it is necessary to add an inductive correction

$$\Phi_{ind} = \omega \alpha_{m,n}(\psi_p) \sin(m\theta - n\zeta + \omega t) \, \frac{I + gq}{nq - m} \tag{6}$$

which is Eq.(7.9) of Ref. [16], with I, g the covariant components of the poloidal/toroidal fields (for details on how the correction can be implemented, see [13]). The inductive correction can be seen as a Doppler shift in the rotating frame of the island, which is consistent with the Stoschus results [17]. Therefore, one can expect that the whole pattern of Φ is shifted, but, due to the common dependence on α, the main geometry of the potential remains unchanged.

We added a paragraph, with the heuristic argument, at the beginning of Page 3, after "... responsible for these flows", plus a comment on the physical context of the simulations, at Page 4, before former Eq.(1). We also discussed the need for an inductive correction, and the expected consequences, at Page 6, before the paragraph on the ambipolar roots.

2. See point above.

3. We simplified the introduction, making it more fusion-oriented. We also clarified the statement about self-organization: in stellarators and reversed-field pinches, magnetic islands occur as the result of a process of self-organization in the strongly coupled, MHD equations. We added a reference to a classic book of Sergio Ortolani and Dalton Schnack on this subject. After the Introduction, we expanded the experimental part on TEXTOR.

4. We added references, as required.

5. We added references to the work by Chapman et al. To the more general problem of the applicability of our results, see Point 1) above.

   The work by Tamain *et al.* on MAST shows that, with a single probe located at a fixed toroidal and poloidal locations, and stepping the phase of the RMP mode from 0 to $60°$, no change in E^r is seen. The measurement is really too poor to conclude anything about the phase dependence of E^r. For the sake of comparison, take into account that the review works by Nicola Vianello on RFX [18, 19], show results from 72 toroidally spaced Langmuir probes, covering the entire torus, in discharges where the phase of a $1/7$ or a $0/1$ mode is varied *continuously*, throughout the discharge.

6. We added references, as suggested by the Referee.

7. We modified the Introduction as suggested by the Referee.

8. On the work by Nardon and Valerie Izzo, see Point 1) above.

9. Maybe the criticism of the Referee is driven by a rushed reading. The $4/1$ island is a side-band of the $3/1$, which is the perturbation applied with the coils. In any case, we clarified this point at Page 3.

10. EIRENE ...

11. Also this issue is maybe due to a rushed reading. Figure 2 is slightly different from the analogous one in Reference 13, and *both ones* are simulations of D_i and D_e on TEXTOR. In any case, we added details in the caption of Figure 2, and explained the slightly different algorithms we used.

12. The radial electric field can be written as

$$E_r = \frac{T_i}{Ze}\frac{\nabla P_i}{P_i} + v_\phi B_\theta - v_\theta B_\phi \tag{7}$$

In our paper, both measurements and simulations refer to the l.h.s. of Eq. (7). The problem with the diamagnetic component arises when one tries to measure E^r from the flow, such as with passive/active spectroscopy. But this is not the case with our paper.

13. Difference figure 3 and 4 ...

14. We added references on the changes in E^r with RMPs.

15. Actually, the work on the electron and ion roots descends directly from the considerations on an analytic form for the potential, and it is an integral part of the algebraic method for finding the ambipolar roots in the stellarator, as shown in Point 1). Since our paper is no more a Letter, there is no need to separate it from the rest of the article.

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